Advanced Math: Notes on Lessons 25-28
David A. Wheeler, 2009-10-11

[Show off slide rules – which are based on logarithms]

Lesson 25: Age Problems/Rate Problems
This one is straightforward.

Lesson 26: Log Form of Exponential / Logarithmic Equation
What are logarithms? They're a special operation that's an "inverse" of exponentiation. This:

$N = b^L \qquad \text{where}\, b > 0, b \neq 1, n > 0$

means the same thing as:

$\log_b N = L \qquad \text{where}\, b > 0, b \neq 1, n > 0$

Why do we have logarithms? Well, one reason is that we need an "inverse" to solve some problems. Addition and subtraction are inverses of each other; multiplication and division are inverses of each other. We can already handle *some* exponentiation problems. When you have to-the-power of a constant value:

$n^3 = 27$

you can use root-of with the same constant value to get the answer:

$\sqrt[3]{n^3} = \sqrt[3]{27} \quad \rightarrow \quad n = 3$

But what happens if the unknown is in the power-of (or root-of) position? I.E., how do we solve this?:

$3^n = 19683$

The answer is, "logarithms". Logarithms are an inverse of the power-of operation. To solve this equation, you take the logarithm of both sides:

$\log_3 3^n = \log_3 19683$
$n = 9$

Here are the two basic rules on logs and exponents:

$\log_b b^n = n \,\text{and}\, b^{\log_b n} = n \qquad \text{where}\, b > 0, b \neq 1, n > 0$

Slide rules are based on logarithms, and until calculators were invented, logarithms were absolutely vital for solving practical problems.

The value "b" notated above is called the "base". Common bases are 10, 2, and *e* (2.71828....). Unfortunately, many documents omit the base and say "it's obvious", where in fact it's *not* obvious; this is due to a long history of different disciplines all using logarithms, but ending up with different conventions for the base. On most calculators, log() uses b=10; if you want base *e*, you use the ln() function instead. ISO standard 31-11:1992 (Mathematical signs and symbols for use in physical sciences and technology) suggests these notations:

- ln(x) means $\log_e(x)$, that is, base=e.
- lg(x) means $\log_{10}(x)$, that is, base=10.
- lb(x) means $\log_2(x)$, that is, base=2.

In class, for now just always include the base (b) when writing a logarithm.

Computing "$\log_{10}$ N" when N is 1 followed by zero or more 0s is easy; the answer is just the number of 0s. So $\log_{10} 1=0$, $\log_{10} 10=1$, $\log_{10} 100=2$, $\log_{10} 1000=3$, and so on.

Logarithms are an example of a function that takes two parameters; on a spreadsheet, LOG(3, 10) is likely to produce $\log_{10}3$. Many calculators can't compute logs to arbitrary bases; we'll discuss later how to compute them in such cases.

Lesson 27: Related Angles / Signs of Trig Functions
This is straightforward. Remember, by convention positive angles go counterclockwise, so negative angles go clockwise. Adding or subtracting 360° doesn't change an angle (presuming that we're only measuring direction, and not the number of rotations to get there); adding or subtracting 180° reverses it.

Lesson 28: Factorial Notation / Abstract Rate Problems
Factorials are easy; n! = n * (n-1) *....*1. So 5! = 5*4*3*2*1. On many computers it's written in functional notation, e.g., FACT(n).

THIS WEEK: Work through lessons 25-28 this week ("Monday – Thursday").

Do home study test #6 this week (covers lessons 21-24), presumably on Friday. Please review your notes before taking it, and memorize what you need first. *Show your work*, so can give partial credit. Have a parent grade it initially (to identify which ones are right or wrong), and put it in the new mailbox on Sunday morning. I intend to do the final grading, so that I can give partial credit (or full credit if it's just an equivalent expression).