

Advanced Math: Notes on Lessons 62-65

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Lesson 62: Abstract Coefficients / Linear Variation

This is an easy lesson about some terminology and notation.

Sometimes, instead of using separate letters for separate constants, we'll use the same letters with different subscripts, e.g., c_1 and c_2 . Don't be confused - these are completely separate values. Watch out; it's easy to miscopy the subscripts.

When you say "y varies linearly with x", that just means that $y=mx+b$; it does not mean that $b=0$ necessarily. When you say "y directly varies with x", that usually means that $y=mx$ (i.e., $b=0$).

Lesson 63: Circles and Completing the Square

The "standard form" of the equation of a circle is:

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

Where x_c and y_c are the x and y coordinate of the center, and r is the radius. This follows directly from the distance formula - a circle is simply all points on a plane that have the same distance from the center.

You can obviously multiply this out and move everything to the left-hand-side, resulting in an equation of the form $\langle \text{stuff} \rangle = 0$. This form is called the "general form"; the general form of a circle is:

$$x^2 - 2xx_c + x_c^2 + y^2 - 2yy_c + y_c^2 - r^2 = 0$$

You can reverse this process - create the standard form from this format - by completing the square of both the x and y groups. Group the x-factors together, and the y-factors together, and complete their squares.

The "standard form" for a circle is much more useful than the "general form", because you can trivially see from the standard form where the center is, and what the radius is... and that it *is* a circle!

Lesson 64: Complex Plane / Polar form of Complex Number / Sums & Products of Complex Numbers

A complex number can be graphed on a rectangular (x,y) coordinate system; the real part is the x (left/right) position, and the imaginary part (the part multiplied by i) is the y (up/down) position. Note that i is up/down on a complex number chart, not left/right. Saxon warns that this can be confusing because \hat{i} means left/right.

You already know about the "rectangular form" and "vector form" of a vector. Since a complex number can be represented as a vector from the origin, complex numbers can be represented both ways too. (I'll just use the Saxon example here.) In particular, given the complex number $2\sqrt{3}+2i$, we can find its angle and distance the same way as a vector. The angle is $\arctan \frac{2}{2\sqrt{3}} = 30^\circ$ and the distance is $\sqrt{(2\sqrt{3})^2 + 2^2} = 4$, so one way to represent this is $4(\cos 30^\circ + i \sin 30^\circ)$. This is an inconveniently long notation, though, so by defining the function $\text{cis } \theta = \cos \theta + i \sin \theta$, we can also say

$4 \operatorname{cis}(30^\circ)$.

The “cis” function is easier to understand if you vary θ from 0° to 360° , and see what $\operatorname{cis} \theta$ computes. [Walkthrough this in class, circling through the angles.]

To add and subtract complex numbers in rectangular form, add/subtract the real parts and add/subtract the imaginary parts.

In general, to add/subtract complex numbers in polar form, you need to convert them to rectangular form (if they have the same angle, or are 180° apart, it's possible, but that's rare).

To multiply complex numbers in rectangular form, cross-multiply them just like you do with polynomials.

To multiply complex numbers in polar form, multiply their absolute values and add the angles.

Lesson 65: Radicals in Trig Equations / Graphs of Log Functions

Just as with other kinds of equations, if there unknowns in a root, you can raise the equation to a power to solve it. E.G., $\sqrt{1-\cos^2\theta}=\cos\theta$ can be squared on both sides, $1-\cos^2\theta=\cos^2\theta$ which simplifies to $2\cos^2\theta=1$. The solutions to this equation are 45° , 135° , 225° , and 315° . Are we done? No.

The problem is that when you raise both sides of an equation to a power, you may lose information that causes spurious answers. In particular, when you square both sides, you eliminate the signs of the original equation, which could be important.

The way to deal with this is to check the “final” solutions by plugging them into the original equation. In this case, the solutions are only 45° and 315° .

The latter part just shows that the graph of the logarithm function and the graph of the exponent function are mirror images of each other; since they are inverse functions of each other, that's to be expected.