Advanced Math: Notes on Lessons 70-73

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Lesson 70: Percentiles and z Scores

When you have a sample from population, the conventional symbols are:

- x $\bar{x}$ ("x bar") - mean of sample
- S_x ("S sub x") - standard deviation of sample

When you have values from a whole population, the conventional symbols are:

- μ ("mu") - mean of population
- σ ("sigma") - standard deviation of population

If your data has the "normal distribution", you can compute the "z-score" of a particular score/value to see how different it was from average. The z-score is just the number of standard deviations a particular score/value differs from the mean, and is calculated as:

$$z\ \ score\ =\ \frac{x-\mu}{\sigma}$$

A z-score can be positive or negative; +2 is "two standard deviations above the mean", and -3 is "three standard deviations below the mean". Finding a z-score is often called "standardizing" or "normalizing", because that single number tells you how far from average the data point is.

From a z-score, you can then compute the "percentile rank". This is the percentage of the population that x is greater than. This is typically how results are reported for standardized tests (for example) - they find your test's specific score, compute the mean and standard deviation for the test population, and then compute the percentile. For our purposes, just use the provided table, which will let you quickly find percentile values from a z-score. Your calculator may have a "cumulative density function" (or something with a similar name) that can make these kinds of calculations (be sure to do the "one-sided" version).

For example: Fred scored 90 on a test, where among the test-takers the average μ was 82 and the standard deviation σ was 4. The z-score of Fred's result = (90-82)/4 = +2... i.e., he was two standard deviations above the average (very good!). Looking up a z-score of 2.0 on the standard normal table shows the entry ".9772". This means that, presuming a normal distribution, his score was better than 97.72% of the test-takers.

By the way, Saxon makes a confusing statement, and shows an equation you really can't use effectively yet. He states that to figure out the percentile you can use this equation:

$$y(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

which is true but not helpful to you yet. This is the general equation of the normal distribution, and from it you can graph the normal distribution. But to find a percentage rank using this equation, you'd need to find the area under it (i.e., integration) - and that's something you don't know how to do yet! So ignore this complicated-looking equation, and use the table (or equivalent calculator buttons).

Lesson 71: The Ellipse (1)

An ellipse is a "squished circle". Instead of a single center, it has two points (singular: focus; plural: foci). An ellipse is the set of points that have the same (distance to focus #1) + (distance to focus #2).

An ellipse has a "major axis" (that goes through the two foci), and a "minor axis" (perpendicular to the major axis, and going through the midpoint between the foci). Conventionally "a" is 1/2 the length of the major axis, and "b" is 1/2 the length of the minor axis.

If the ellipse's major axis is horizontal, the standard form is:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

If the ellipse's major axis is vertical, the standard form is:

$$\frac{x^2}{b^2}+\frac{y^2}{a^2} = 1$$

This is an easy lesson about some terminology and notation.

Lesson 72: One Side plus Two Other Parts, Law of Sines

If we know the length of a triangle, and two other values (angles or lengths), we can solve all the lengths and angles. You have to have at least one length; with only fixed angles, you can scale the triangle to arbitrarily larger/smaller sizes.

If you have a right triangle, you already know an angle, so you just need a length and something else. From there, you already know how to use trigonometry to find the rest. But what if you don't have a right triangle (aka an oblique triangle)? A useful approach is to use the "law of sines", which is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Where A, B, and C are angles, and a, b, and c are the lengths of the sides opposite those angles.

Beware: If you use arcsine to find a value, there are normally two possible angles that solve an arcsine, but a calculator will only show one of them (the first quadrant answer). Often you can just look at the triangle to see if the angle is less than 90 degrees, or more than 90 degrees.

Lesson 73: Regular Polygons

A regular polygon (all sides congruent, all interior angles congruent) of any number of sides can be inscribed in a circle (i.e., a circle can cover all the polygon's vertices). The circles' center will also be the regular polygon's center. If you draw segments from the circle's center to each of the polygon's vertices, you'll create a bunch of isosceles triangles. The figures in the book are good - in fact, I don't see how to improve on them.

Winner of the most obscure mathematical term: apothem. I didn't even remember that one. Anyway, The "apothem" is the altitude of any of those isoscles triangles, that is, the distance from the circle/polygon's center to the base segment of one of those triangles. The apothem bisects the isosceles triangle, creating two triangles; this means that all the tools you have that work on triangles (and you know many) work with them. If you want to inscribe a circle inside the polygon (the *opposite* of what we started with - this circle will cover the polygon's sides' *midpoints*), the apothem is its radius.